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Towards Uncertain Portfolio Selection

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1 Introduction

Portfolio selection is concerned with optimal allocation of money to a number of securities. Textbooks tell us that we should use Markowitz methodology to select the portfolio. However, when we did surveys on portfolio selection from professional investors, it was found that none of the investors surveyed selected portfolios according to Markowitz methodology. Why is that? The paper will review Markowitz methodology first and then discuss why investors do not use the methodology. Next, we will show that we cannot use fuzzy set theory to select portfolios either. After that we will introduce a new portfolio selection theory which we call uncertain portfolio selection. We will also introduce the difference of the uncertain portfolio theory from traditional portfolio theory and tell when we should use uncertain portfolio theory. The future research problems in the area of uncertain portfolio selection are also mentioned.

2 Markowitz Methodology

The security return is expressed by the rate of return which is defined as

$$\frac{\text{receipt} - \text{expenditurn}}{\text{expenditure}}.$$

Without considering transaction costs, taxes and stock splits, the security return can also be defined as

$$\frac{\text{ending price of a security} - \text{beginning price+cash dividend}}{\text{beginning price}}.$$

According to Markowitz [16], security returns are regarded as random variables. The investment return and risk of a portfolio are measured by the portfolio's expected return and the variance, respectively, which are calculated using the historical return data of the candidate securities. Then the investors should pursue the maximum expected return at a given specific level of investment risk. The standard formulation of Markowitz model is as follows:

$$\begin{cases} \max E[x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n] \\ \text{subject to:} \\ \qquad V[x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n] \le c \\ \qquad x_1 + x_2 + \cdots + x_n = 1 \\ \qquad x_i \ge 0, \quad i = 1, 2, \cdots, n \end{cases} \tag{1}$$

where x_i are the investment proportions in securities i, ξ_i the random returns of the securities i, $i = 1, 2, \cdots, n$, respectively, E and V denote the expected value operator and variance operator of random variables, respectively, and c is the maximum variance level the investors can tolerate.

In Markowitz model, expected return of a portfolio is obtained by calculating the average return of the portfolio using the N samples of returns of each component security. The N samples of each component security are the N historical returns of the security. Table 1 shows the computation of the expected return of a portfolio with three candidate securities using twelve historical monthly returns

Table 1: Computation of a Portfolio's Expected Return and Variance according to the Component Securities' Historical Returns

Month j	r_{1j}	r_{2j}	r_{3j}	$R_j = 0.2r_{1j} + 0.2r_{2j} + 0.6r_{3j}$ (use SUMPRODUCT)
Dec. 2016	0.2011	0.2981	0.3248	0.2947
Jan. 2017	0.1281	0.1179	0.6418	0.4343
Feb. 2017	-0.0305	-0.0421	0.0598	0.0214
Mar. 2017	0.0456	0.1789	0.1797	0.1527
Apr. 2017	0.1286	-0.2355	0.7050	0.4016
May 2017	0.1048	0.1474	-0.0836	0.0003
Jun. 2017	-0.0849	0.2463	0.3543	0.2449
Jul. 2017	0.2264	0.0305	-0.0808	0.0029
Aug. 2017	0.3706	0.1455	-0.0255	0.0880
Sept. 2017	-0.0162	-0.0222	0.2963	0.1701
Oct. 2017	0.0176	0.2517	-0.2678	-0.1068
Nov. 2017	-0.2156	-0.3124	-0.2329	-0.2453
Mean	(use AVERAGE)			0.1216
Variance	(use VAR.P)			0.0376

of the securities as an illustration. In Table 1, the symbol R_j denote the j-th sample of the return of the portfolio consisting of 20% security 1, 20% security 2 and 60% security 3 for $j, j = 1, 2, \cdots, 12$, respectively, and r_{ij} represent the j-th monthly returns of the securities i, $i = 1, 2, 3, j = 1, 2, \cdots, 12$, respectively. With the existing solver, for example, Microsoft Excel, we can use "SUMPRODUCT" to calculate the 12 samples of the portfolio return, i.e.,

$$R_j = 0.2r_{1j} + 0.2r_{2j} + 0.6r_{3j}, \quad j = 1, 2, \cdots, 12$$

where the weights are the money proportions of the individual securities in the portfolio. Then with the data R_j, use the function "AVERAGE" to easily get the expected value of the 12 samples of the portfolio return. In general, the expected value of a stochastic portfolio return $E[\xi]$ is computed as follows:

$$e = E[\xi] = \sum_{j=1}^{N} R_j/N = \sum_{j=1}^{N}(x_1r_{1j} + x_2r_{2j} + \cdots + x_nr_{nj})/N.$$

Variance of a stochastic portfolio return can be obtained according to the definition of variance, i.e.,

$$V[\xi] = E[(\xi - e)^2] = \sum_{j=1}^{N}(R_j - e)^2/N.$$

For example, the variance value of the stochastic return of the portfolio in Table 1 is obtained as follows:

$$V[\xi] = \sum_{j=1}^{12}(R_j - 0.1216)^2/12 = 0.0376.$$

Similarly, we can also use the existing tools to get the variance. For example, for each index $j, j = 1, 2, \cdots, 12$, with the solution tool of Microsoft Excel we can first use "SUMPRODUCT" to calculate each sample of the portfolio return R_j. Then with all the twelve R_j data, use the function "VAR.P" to easily get the variance. Please see Table 1.

Having known the way to calculate the expected values and variances of portfolio returns, investors can then use the existing tools to solve the problem of the model (1) to find the optimal portfolio. For example, we can use the command "Solver" in the menu "Tool" in Microsoft Excel to solve the model (1). When calculating the expected value and variance of the portfolio return, use the method introduced above. The difference is that in the above introduction, the investment proportions in the three component securities are determined, while when computing the expected value and variance of the portfolio in the model (1), the decision variables $x_1, x_2, \cdots, x_n$ replace the determinate proportions.

3 Do Investors Really Use Markowitz Methodology?

From the above review we can see that Markowitz methodology selects the portfolio completely based on the historical returns of the securities. But do investors really do so in practice?

To find the answer, I did deep surveys by face-to-face investigation to 15 fund managers who are professional investors and know Markowitz methodology well. In the surveys, I first asked,

"Do you select portfolio completely based on the past returns of the securities?"

None of them said yes. Next I asked,

"Then how do you select portfolio?"

Table 2: Information Used by the Fifteen Managers in Estimating Stock Returns

Manager No.	Historical Return	Company's Profit & Prospect	Government Policies	Market News	Transaction Volume
1	√	√	√	√	√
2		√	√		
3		√	√	√	
4		√	√	√	
5		√	√		
6	√	√	√		
7	√	√	√		√
8		√	√	√	√
9	√	√	√	√	
10	√	√	√	√	√
11	√	√	√	√	√
12		√	√		√
13		√	√		√
14		√	√	√	
15		√	√		

Though their descriptions are not quite the same, they all estimate the securities' returns first and then select the portfolio. The information they used for estimation of the securities' returns are of variety. Regarding the role of past return data, five of the interviewees stated that the historical data would be used as one reference to judge the tendency of the securities' returns and one of the interviewees said that the past returns would be used to estimate the stability of the returns.

For example, for the fund manager 1, he said he would consider the historical returns of the stocks to feel the variation level of the concerned securities' returns. To predict the stocks' prices, he would carefully analyze the stock companies' profits and the prospect of the industries the companies are in. He used the expression of "studying financial statements and analyzing performances of the underlying companies", and stressed that the underlying companies should be from the "promising industries" and should be "competitive" in the industries. Especially, He stressed that when analyzing the stock companies' performances, he not only analyzed the financial statements of the companies but also conducted on-site investigation of the companies. Besides, he would keep an eye on the government policies that are related to the industries and the companies, and would also pay attention to the market theme and the market news. He gave me an example to understand it. For example, the Chinese government proposed the Belt and Road Initiative. Then the companies involved in the initiative may have good development in the future. Besides, the stock market is in favor of the initiative, which in turn can further push the stock prices of the relative companies to increase quite a lot. In addition, he stated that he would pay much attention to the transaction volume of the stocks, especially, the transaction volume of the big dealers because their actions have great effect on the stocks' prices. For the fund manager 2, he focused on the stock companies' performances and the prospect of the industries the companies are in. He used the expression of "analyzing fundamentals of the underlying companies" and emphasized that the companies should be from the "growing industries" and should be "competitive" in the industries. He also stressed the importance of on-site investigation of the companies in estimating the companies' performances. Besides, he is also sensitive to the government policies. I summarize the information they used in estimating the securities' returns in Table 2.

From Table 2 we can see the following facts:

(1) Among the 15 fund managers, 9 managers do not care historical returns at all.

(2) Even among the 6 managers who care historical returns, they also care company's profit and prospect as well as government policies. In addition, some managers care market news and stock transaction volume too.

Thus, from the surveys we get that no one produces distribution functions of the securities' returns *completely* based on the past return data. In other words, they do not use Markowitz methodology to get the return distribution functions of the securities. So we say no one among them uses Markowitz methodology in real practice.

4 Why Do the Investors Not Use Markowitz Methodology?

In the above section, we have shown that when making investments, the investors use their estimations of the security returns rather than complctely the historical return data. Before answering why the investors do not use Markowitz methodology, let us first give a simple example to show how to get the distribution functions of the security returns according to the experts' (or investors') estimations. More information about the method of obtaining the return distribution functions based on the experts' (or investors') estimations can be found in paper [5] and book [15]. The interested readers can refer to them.

Example 1. Suppose when an investor is asked "What do you think is the security return?", he answers, "I think the minimum return of the security will be -0.1, and the chance of the return being less than 0 will be 80%, but the maximum return of the security should be 2." Then from the investor's estimations, we get the following three statements:

(1) I believe that the return being greater than -0.1 will happen with chance 1, which can further be translated into the form that I believe the return being less than -0.1 will happen with chance 0.

(2) I believe that the return being less than 0 will happen with chance 0.8.

(3) I believe that the return being less than 2 will happen with chance 1.

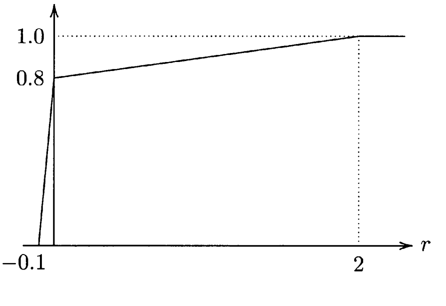

Figure 1: Belief degree function of the security return got from the estimations in Example 1.

We can see that the estimations express the chance levels with which the investor believes the indeterminate return events will happen. From the first statement we get one point $(-0.1, 0)$. From the second statement we get another point (0, 0.8), and from the third statement we get one more point (2, 1). Connect the three points, then we get a line. This line is the distribution function of the security return got from the estimations. We call it *belief degree function* of the security return. Please see Fig. 1. With the belief degree function, we can know the chance level of the return lower than a given point. For example, with the belief degree function in Fig. 1, we can know that the investor believes the chance of the security return being less than 1 is 0.9.

Some people feel that we can still treat the belief degree functions of the returns, e.g., the belief degree function in Fig. 1, as probability distributions. However, the finding of Kahneman and Tversky (1979) points out that people give too much weight to unlikely invents, which implies that investors usually estimate much wider range of values than the returns on the securities actually take. A lot of surveys in portfolio investments have confirmed the phenomenon too. Then what if we still model belief degree functions of the returns by probability theory? Let us study an example.

Example 2. Suppose investors have 80 securities with independent identical return distributions. Suppose also the security returns actually distribute evenly on $(-0.08, -0.02)$. Since the maximum return of each individual security is -0.02, it is easy to get that

$$\Pr\{\text{"return of the investment evenly in the 80 securities"} \leq 0\} = 1.$$

That is, it is certain that the portfolio return is less than 0. However, as many factors change greatly, investors must estimate the security returns. In real life, it is rare that people can estimate the returns exactly the same as in the real case. Suppose the investors' belief degree function for each individual security return is the one shown in Fig. 1. If the belief degree functions of the returns are still treated as probability distributions, we can get by simulation that

$$\Pr\{\text{"return of the investment evenly in the 80 securities"} \leq 0\} = 0.0003 \approx 0.$$

It is seen that by inappropriately using probability theory, a certain event is judged to be an almost impossible event. Suppose the return of the risk free security is 0.02, and the investors are considering how to allocate their money among the risky securities and the risk free security. The investors are cautious and set a strict risk control requirement that the chance of the portfolio return being lower than 0 must be equal to or smaller than 0.1%. Then we can see that in order to obtain the maximum expected investment return with the risk control requirement, the investors should allocate all their money to the 80 risky securities rather than in the risk free security because

$$\Pr\{\text{"return of the investment evenly in the 80 securities"} \leq 0\} < 0.001,$$

and expected return of the investment evenly in the 80 risky securities is 0.16 which is much higher than the risk free return rate of 0.02. However, if the investors followed the decision, they would suffer at least a loss of -0.02 because the maximum real return of each security is only -0.02. It is seen that when the distribution functions obtained from investors' estimations are not close enough to cumulative frequencies, treating the distribution functions as probability distributions is inappropriate and can lead the investors to great loss. So the investors cannot use Markowitz methodology.

5 Can We Model the Estimated Returns by Fuzzy Numbers?

Other than probability distribution, can we model the humans' estimations of security returns in other way? Some people think fuzzy numbers can be used to model investors' estimations of security returns. Do investors estimate returns in fuzzy way? Can investors use fuzzy set theory to select portfolio? Let me use an example to answer these questions.

In fuzzy set theory, membership function is the basic concept, possibility is the basic measure, and each fuzzy number should have a membership function. According to fuzzy set theory, the relationship between membership function and possibility measure is as follows,

$$\mathrm{Pos}\{\xi \in B\} = \sup_{x \in B} \mu(x), \tag{2}$$

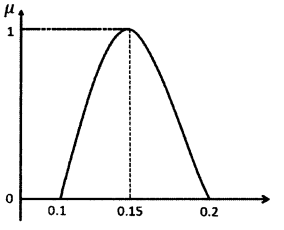

Figure 2: Membership function of a security turn.

where ξ is a fuzzy number, μ the membership function, and B a real number set.

Example 3. Let us first assume we can use fuzzy numbers to describe investors' estimations of the security returns. Then each fuzzy number should have a membership function. Suppose that the membership function of a security return is as shown in Fig. 2. From Fig. 2 and the equation (2) we can easily infer that

$$\text{Pos}\{\text{"The security return is exactly 0.15"}\} = 1. \tag{3}$$

$$\text{Pos}\{\text{"The security return is not exactly 0.15"}\} = 1. \tag{4}$$

It is easy to get from equations (3) and (4) that

$$\text{Pos}\{\text{"The return is exactly 0.15"}\} = \text{Pos}\{\text{"The return is not exactly 0.15"}\}. \tag{5}$$

Let us study the meaning of equations (3) and (5). Equation (3) says that the security return is exactly 0.15, neither a little more nor a little less, is with maximum possibility value of 1. But what a coincidence it should be in order that the security return is exactly 0.15. Judging from common sense, people will believe that the chance of "the return being exactly 0.15" should be almost zero, which implies that humans do not think in fuzzy way. Equation (5) indicates that the event of "the return being exactly 0.15" and the event of "the return not being exactly 0.15" will occur equally possibly. But "the return not being exactly 0.15" will surely occur with much higher chance than "the return being exactly 0.15". Suppose there is a bet: You get \$100 if the security return is exactly 0.15 and you pay \$100 if the security return is not exactly 0.15. Will you accept such a bet? No one will because we will judge that the chance of the return not being 0.15 is by far much higher than the chance of the return being exactly 0.15. That bet is much too unfair. This means we cannot accept equation (5). In other words, people do not estimate in fuzzy way. Furthermore, book [14] has pointed out by illustration that theoretical foundation of fuzzy set theory has flaws. Therefore, fuzzy set theory is not suitable for modelling estimations of security returns. Nor is it suitable for dealing with the portfolio selection problem.

6 Towards Uncertain Portfolio Selection

To find a suitable tool to model humans' estimations of the security returns, we turn to uncertainty theory. Uncertainty theory was founded in 2007 [12] and subsequently studied by many scholars. Now it has been developed to be a branch of axiomatic mathematics [14] to model humans' estimations towards indeterminate events. The uncertain measure is interpreted as the chance with which the person believes an uncertain event may happen, and uncertainty distribution is called belief degree function in practice. The four axioms of the uncertainty theory are given below. For uncertainty theory, please refer to [15].

Definition 1 Let Γ be a nonempty set, and L a σ-algebra over Γ. Each element $\Lambda \in L$ is called an event. A set function $\mathcal{M}\{\Lambda\}$ is called an uncertain measure if it satisfies the following three axioms [12]:

(i) (Normality) $\mathcal{M}\{\Gamma\} = 1$.

(ii) (Duality) $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$.

(iii) (Subadditivity) For every countable sequence of events $\{\Lambda_i\}$, we have

$$\mathcal{M}\left\{\bigcup_{i=1}^{\infty}\Lambda_i\right\} \le \sum_{i=1}^{\infty}\mathcal{M}\{\Lambda_i\}.$$

The triplet $(\Gamma, L, \mathcal{M})$ is called an uncertainty space.

(iv) (Product Axiom) [13] Let $(\Gamma_k, L_k, \mathcal{M}_k)$ be uncertainty spaces for $k = 1, 2, \cdots$ The product uncertain measure $\mathcal{M}$ is an uncertain measure satisfying

$$\mathcal{M}\left\{\prod_{k=1}^{\infty}\Lambda_k\right\} = \bigwedge_{k=1}^{\infty}\mathcal{M}_k\{\Lambda_k\}$$

where Λ_k are arbitrarily chosen events from L_k for $k = 1, 2, \cdots$, respectively.

Definition 2 [12] An uncertain variable is defined as a function from an uncertainty space $(\Gamma, L, \mathcal{M})$ to the set of real numbers such that for any Borel set of B of real numbers, the set

$$\{\xi \in B\} = \{\gamma \in \Gamma | \xi(\gamma) \in B\}$$

is an event.

Definition 3 [12] The uncertainty distribution Φ of an uncertain variable ξ is defined by

$$\Phi(t) = \mathcal{M}\{\xi \le t\}$$

for any real number t.

Let us reconsider the above Example 2. If the belief degree functions of the 80 risky security returns shown in Fig. 1 are regarded as uncertainty distributions, according to the uncertainty theory, we can get that

$$M\{\text{"return of the investment evenly in the 80 securities"} \le 0\} = 0.8. \tag{6}$$

A certain event is not judged to be an almost impossible event. We see that the chance of the portfolio investment return evenly in the 80 risky securities being lower than 0 is as high as 80%. Though the chance of 80% does not reach to the true value 100%, the difference is from the errors in the estimations. When there are errors in estimations, uncertainty theory does not further magnify the estimation errors, while probability theory magnifies the errors. With the result of (6), the investors now should allocate all their money to the risk free security. Then they can gain a return of 2% with certainty. Furthermore, we can see that even if the investors are high risk tolerable people who set the tolerable chance of the investment return being lower than 0 to be as high as 60% in order to trade for high expected return, using uncertainty theory the investors will still choose to invest in the risk free security instead of the 80 risky securities, which will let the investors avoid the loss.

We propose that a suitable tool for a decision making problem should be self-consistent and in the meantime be able to solve this type of problem best among other tools. Uncertainty theory meets the requirement in solving portfolio selection problem. Using uncertainty theory as the tool, Huang initiated in 2010 an uncertain portfolio theory [3] in which a systematic selection models have been proposed and studied. In uncertain portfolio theory, uncertain variables are used to model the securities' returns, and belief degree functions got from experts' estimations are regarded as uncertainty distributions of the security returns. For example, let ξ_i denote the i-th security returns estimated by experts for $i = 1, 2, \cdots, n$, respectively. In uncertain portfolio selection, ξ_i are treated as uncertain variables. If we adopt the chance of the portfolio investment return failing to reach a preset threshold return level to measure the investment risk, we have the following uncertain mean-chance model [3]

$$\begin{cases} \max E[x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n] \\ \text{subject to:} \\ \qquad \mathcal{M}[x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n \le c] \le \alpha \\ \qquad x_1 + x_2 + \cdots + x_n = 1 \\ \qquad x_i \ge 0, \quad i = 1, 2, \cdots, n \end{cases} \tag{7}$$

where E denotes the expected value operator, $\mathcal{M}$ the uncertain measure of uncertain variables, and c is threshold return level and α the tolerable chance level. The problem is solved by using uncertainty theory. The method can be found in book [3].

So far, a spectrum of uncertain portfolio selection models have been proposed and discussed besides uncertain mean-chance model. If we use variance of the portfolio return as the investment risk, we have the uncertain mean-variance model [3]. If we use semivariance of the portfolio return as the investment risk, we have the uncertain mean-semivariance model [5]. If we define risk curve as the investment risk, we have the mean-risk model [4]. If we adopt risk index as the investment risk, we have the mean-risk index model [6]. Besides, if we take into account background risk in portfolio selection, we have the mean-chance background risk model [7]. Uncertain portfolio selection has attracted more and more scholars' interest nowadays. For example, the extension of the uncertain mean-variance models [1, 20] and mean-semivariance model [2] have been studied. In addition, mean-semiabsolute deviation selection model [11] and mean-semiabsolute deviation adjustment model [17] have been proposed, and mean-risk model considering background risk has been researched [19], etc.

If we define stochastic portfolio selection to be a methodology for selecting portfolio based on frequencies and probability theory, then uncertain portfolio selection is defined to be a methodology for selecting portfolio based on belief degree functions and uncertainty theory. There are three differences of uncertain portfolio selection from stochastic portfolio selection. First, the input is different. The input of stochastic portfolio theory is the frequencies of the past return data, while the input of uncertain portfolio selection is the uncertainty distributions got from humans' estimations. Second, stochastic portfolio selection and uncertain portfolio selection apply different mathematical tools. The former uses probability theory while the latter uses uncertainty theory. Rooted from the second difference, the third difference is that the mathematical properties and the solution methods of the optimal selection problems in two theories are different.

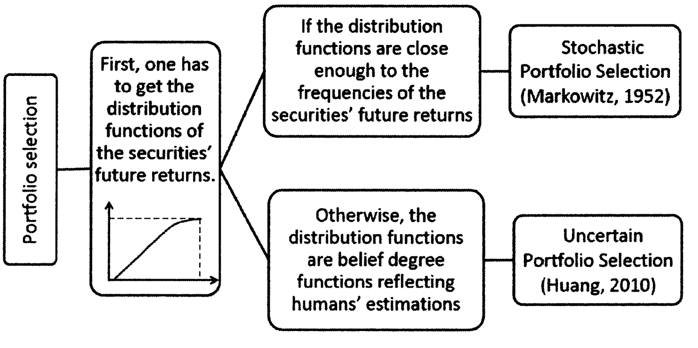

Figure 3: When to use uncertain portfolio selection theory?

Perhaps the readers would like to ask when we should use uncertain portfolio selection and what future research work can be done in uncertain portfolio selection. I would answer in this way. In order to select a portfolio, one first needs to get the distribution functions of the candidate securities' returns. If you believe the distribution functions are close enough to the frequencies of the securities' future returns, you should use stochastic portfolio selection. Otherwise, you should treat the obtained distribution functions as uncertainty distributions (i.e., belief degree functions) and use uncertain portfolio selection (Please see Fig. 3). Uncertain portfolio selection has been initiated not very long ago. Generally speaking, problems arose in stochastic portfolio selection are worth researching in uncertain portfolio selection. The detailed discussion of it can also be found in paper [9].

7 Summary

Uncertain portfolio selection is a new branch of portfolio theory which is defined to be a methodology for selecting portfolio based on belief degree functions and uncertainty theory. When selecting portfolio, investors must first obtain the distribution functions of the candidate securities' returns. If the distribution functions are not believed to be close enough to the frequencies of the securities' future returns, the distribution functions have to be treated as belief degree functions which reflect humans' estimations of the securities' future returns. In this case, uncertain portfolio selection should be adopted. Since in real life investors usually choose their portfolios from only those stocks whose companies' profits or other factors that affect faith in the companies change favourably and greatly, the stock returns cannot be reflected completely by historical return data and have to be given by investors' estimations. It is rare that humans' estimations can be fairly close enough to the securities' future frequencies. Therefore, uncertain portfolio selection is a prospective research area and there is great room for research in this field. Generally speaking, what have been studied in traditional portfolio selection are worth researching on uncertain portfolio selection.

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